The photon structure and exclusive production of vector mesons in $\gamma\gamma$ collisions

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Abstract. The process of exclusive vector meson production, $\gamma \gamma \rightarrow J/\psi \rho^0$, is studied for almost real photons. This process may be reduced to the photoproduction of J/ψ off the ρ^0 meson. We discuss the possibility of extracting the gluon distribution of ρ^0 and of the photon from such a measurement. Predictions are also given for the reaction $e^+e^- \rightarrow e^+e^-J/\psi\rho^0$ for various e^+e^- cms energies typical for LEP and for the future linear colliders.

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The successful operation of LEP2 has given a significant boost for extensive studies of the photon structure – both theoretically and experimentally (see e.g. [1,2]). We may expect further progress in this field after the next generation of high energy linear colliders have started. However, up to now, the existing results for the photon structure have rather limited accuracy. They are based mainly on the inclusive data, i.e. the total $\gamma \gamma^*$ cross section. It should therefore be useful to supplement this classical approach with some other independent methods of measurement of the partonic content of the photon. In fact, the Ryskin process [3] of the elastic vector meson photoproduction at HERA may serve as a good example of such an alternative way of extracting the gluon distribution of the proton [4]. The cross section for the exclusive reaction $\gamma\gamma \rightarrow J/\psi V$ is expected to be dominated by the light vector meson V components of the photon. Thus, the exclusive reaction $\gamma \gamma \rightarrow J/\psi V$ should closely resemble the analogous well-known one: $\gamma p \rightarrow J/\psi p$. Both have the advantage of directly testing the gluon distribution with enhanced sensitivity to the x dependence at low x. Therefore, we shall analyze in detail the exclusive vector meson production $\gamma \gamma \rightarrow J/\psi V$ and show that, indeed, it may probe efficiently the gluon distribution of $V = \rho^0$, ω and ϕ . Hence, it may also constrain the gluon distribution of the photon. This new proposed measurement, when combined with HERA data, may also test the universality of the Regge behavior in the exclusive vector meson production off different hadrons. Although such reactions of two photon have already been studied, in particular in [5,6], the connection of the amplitude to the gluon distribution has never been exploited before. Finally, in order to estimate the feasibility of this measurement we shall also compute the corresponding e^+e^- cross sections.

In most of the successful models of the photon structure [7–9] it is assumed that the photon wave function (in the strongly interacting sector) is represented by two significantly different sets of configurations: a hadronic-like one, i.e. virtual light vector mesons V, and a perturbative one, corresponding to $q\bar{q}$ or $q\bar{q}g$ systems with transverse momenta in the perturbative domain. For the real photon, the vector meson configurations dominate the photonproton cross section (the vector meson dominace model). The same property holds true for the $\gamma\gamma$ total cross section as well (see e.g. [9]). It is therefore legitimate to assume that in an exclusive process with a light vector meson in the final state, the dominance will be even more pronounced. The perturbative configurations, less important for the total cross section, will tend to produce hadronic systems with higher invariant masses, not contributing to the selected exclusive channel. Thus, the exclusive process $\gamma \gamma \rightarrow J/\psi V$ occurs predominantly through the vector meson component V of one of the incoming photons. This means that the scattering amplitude $\mathcal{M}(\gamma\gamma \to J/\psi V)$ will be well approximated by the amplitude $\mathcal{M}(\gamma V \to J/\psi V)$ multiplied by the photon-meson coupling $\alpha^{1/2}q_V$.

It was pointed out by Ryskin [3] that the elastic J/ψ photoproduction off the proton at the γp cms energy W may probe the gluon distribution of the proton, $xG^p(x, Q^2)$, for $x = M_{J/\psi}^2/W^2$ and $Q^2 = M_{J/\psi}^2/4$. Namely, the obtained differential cross section for t = 0,

$$\frac{\mathrm{d}\sigma_{\gamma p \to J/\psi p}}{\mathrm{d}t} (W^2, t=0) = \frac{16\pi^3 [\alpha_{\mathrm{s}}(M_{J/\psi}^2/4)]^2 \Gamma_{ee}^{J/\psi}}{3\alpha M_{J/\psi}^5} \left[x G^p(x, M_{J/\psi}^2/4) \right]^2, \quad (1)$$

depends on the square of $xG^p(x, M^2_{J/\psi}/4)$, which allows for the direct measurement of this quantity with enhanced sensitivity. This concept has proven to be extremely succesful, and it strongly stimulated the experimental efforts at HERA [10,11]. Since the first paper [3] significant theoretical progress in this subject has also been made [12,13]. A set of important corrections to the leading result (1) was isolated and studied in detail. We shall present these, following the discussion in [4, 14]. In particular, it was shown that relativistic corrections may be absorbed into the constituent mass of the charmed quark, $m_c = M_{J/\psi}/2$ [4,15]. Formula (1) takes into account only the dominant imaginary part of the complex amplitude $\mathcal{M}(\gamma p \to J/\psi p)$. The estimate of the smaller real part of $\mathcal{M}(\gamma p \to J/\psi p)$ may be obtained in a standard way from the Regge model. The additional enhancement due to the real part of the matrix element may be described by a multiplicative correction factor:

$$C_{rp} \simeq 1 + \frac{\pi^2 \lambda^2}{4},\tag{2}$$

with $1 + \lambda$ defined as the (local) pomeron intercept:

$$\lambda = \frac{\partial \log[xG(x,Q^2)]}{\partial \log(1/x)}.$$
(3)

The QCD NLO corrections to the $\gamma J/\psi$ impact factor [4] introduce an additional factor of

$$C_{\rm NLO} \simeq 1 + \frac{\alpha_{\rm s}(M_{J/\psi}^2/4)}{2}.$$
 (4)

It has been pointed out that there are non-negligible corrections coming from the fact that the parton distribution entering (1) should be in fact off-diagonal [16]. A detailed study [17] showed that in order to account for this effect the cross section should be multiplied by a factor

$$C_{\rm off} \simeq 1.2. \tag{5}$$

The value of C_{off} is governed mainly by the degree to which the parton distribution is off-diagonal in the production process, and it should not be too sensitive to differences in the structure of ρ^0 and the proton and to the choice of x. It seems that the effects of rescattering of the $c\bar{c}$ pair and of the exchanged gluon transverse momenta partially cancel each other [4]. The overall effect is small; hence, including these corrections is not necessary to maintain the compatibility between the theoretical and the experimental results [14].

For the sake of lowering statistical errors it is favorable to study the total cross section $\sigma_{\gamma p \to J/\psi p}(W^2)$. Due to the (approximate) exponential decrease of $d\sigma/dt$:

$$\frac{\mathrm{d}\sigma_{\gamma p \to J/\psi p}}{\mathrm{d}t}(W^2, t) \simeq \frac{\mathrm{d}\sigma_{\gamma p \to J/\psi p}}{\mathrm{d}t}(W^2, t=0) \exp(B_{J/\psi p}t),\tag{6}$$

with $B_{J/\psi p}$ equal to 5 GeV⁻² and very weakly depending on W^2 [10], one may write

$$\sigma_{\gamma p \to J/\psi p}(W^2) = \frac{1}{B_{J/\psi p}} \frac{\mathrm{d}\sigma_{\gamma p \to J/\psi p}}{\mathrm{d}t}(W^2, t=0).$$
(7)

Recent refined studies [4, 13, 14] have shown that the theoretical results agree very well with the data from HERA [10]. Of course, it is straightforward to generalize formula (1) for the case of production of J/ψ off ρ^0 , ω and ϕ . There are two different approaches to model the structure of vector mesons. The first one is based on the expected, approximate similarity between the spatial wave function of the pion and the light vector mesons, which implies $xG^V(x,Q^2) \simeq xG^{\pi}(x,Q^2)$ [8]. Another point of view is presented in [7], in which the differences between vector mesons and pions (e.g. spin, lifetime) are underlined and the parton distributions of $V = \rho^0$, ω and ϕ are fitted to describe the experimental results for the photon structure. We shall include in our analysis both possibilities.

Certainly, there arise subtleties. First, there exist no direct experimental results for $xG^V(x, Q^2)$ and the existing parameterizations are based on models. Therefore, the results we shall obtain will be uncertain. However, this analysis is more aimed to estimate the feasibility of the measurement than to provide a precise answer. Besides, it makes the proposed measurement the first one to consider to probe directly the gluonic content of the light vector mesons. Second, the value of $B_{J/\psi\rho}$ for the exclusive production off the light vector mesons has never been measured. Fortunately, it may be constrained by theoretical considerations on the basis of indirect data. With a reasonable accuracy the B coefficients in the studied processes $(pp \to pp, \gamma p \to Vp, \gamma^* p \to Vp \text{ and } \gamma p \to J/\psi p)$ may be described as additive combinations of characteristic components b_i for the hadrons h_i taking part in the process, i.e., $B_{ij} = b_i + b_j$. In the dipole picture of the diffractive scattering, these components may be related to the mean squared sizes of the contributing dipoles (see e.g. [18]). For energies $50 \,\mathrm{GeV} < W < 100 \,\mathrm{GeV}$ the following values of B has been reported: for elastic pp scattering values of *B* has been reported: for elastic *pp* scattering $B_{pp} \simeq 10{-}12 \,\text{GeV}^{-2}$, for ρ^0 photoproduction at HERA $B_{\rho p} \simeq 11 \,\text{GeV}^{-2}$, decreasing with Q^2 when the photon is virtual [11], and finally for J/ψ photo- and electroproduction off the proton $B_{J/\psi p} \simeq 5 \,\text{GeV}^{-2}$. Taking into acount that $b_{J/\psi} \ll b_p$, we arrive at the conclusion that $b_p \simeq b_\rho \simeq B_{J/\psi \rho} \simeq 5.5 \,\text{GeV}^{-2}$. Due to the experimental errors for the B coefficients (typically $\pm 1 \,\text{GeV}^{-2}$) and the crudeness of the estimation of the latter result, we adopt $B_{J/\psi\rho} \simeq (5.5 \pm 1.0) \,\text{GeV}^{-2}$. Finally, the corrections due to rescattering may differ for production off the proton and off the light vector mesons. This difference should not be significant for the result, because the correction is small anyway; moreover, as suggested by the relation $b_p \simeq b_o$ the sizes of the proton and ρ^0 are similar. So, for the estimate of the differential $\gamma\gamma \rightarrow J/\psi\rho^0$ cross section we shall use the following formula:

$$\frac{\mathrm{d}\sigma_{\gamma\gamma\to J/\psi\rho}}{\mathrm{d}t}(W^2,t) = C_{\mathrm{off}}C_{\mathrm{NLO}}C_{rp}\alpha g_{\rho}^2 \frac{16\pi^3 [\alpha_{\mathrm{s}}(M_{J/\psi}^2/4)]^2 \Gamma_{ee}^{J/\psi}}{3\alpha M_{J/\psi}^5} \times \left[xG^{\rho}(x,M_{J/\psi}^2/4)\right]^2 \exp(B_{J/\psi\rho}t),$$
(8)

where W denotes the $\gamma\gamma$ cms energy, $x = M_{J/\psi}^2/W^2$ and $g_{\rho}^2 = 0.454$.

The real opportunity to study high energy photon– photon collisions is provided so far only in e^+e^- colliders mainly at LEP2¹. There, a large fraction of events may be described in terms of a factorizing $\gamma\gamma$ cross section and known photon fluxes in the colliding leptons [1]:

$$d\sigma_{e^+e^- \to e^+e^- X}$$

$$= f_{\gamma/e}(z_1, Q_1^2) f_{\gamma/e}(z_2, Q_2^2) \sigma_{\gamma\gamma \to X}(W^2) dQ_1^2 dQ_2^2 dz_1 dz_2,$$
(9)

where $W^2 = z_1 z_2 s_{ee}$, s_{ee} is the cms leptonic collision energy squared, and the flux factor for the photon carrying the fraction z of the incoming electron and the virtuality Q^2 reads

$$f_{\gamma/e}(z,Q^2) = \frac{\alpha}{2\pi} \left[\frac{1}{Q^2} \frac{1 + (1-z)^2}{z} - \frac{2zm_e^2}{Q^4} \right], \quad (10)$$

where m_e is the electron mass. The lower limit for Q_i^2 follows directly from the kinematics of the process:

$$Q_{\min}^2 = \frac{m_e^2 z^2}{1-z}.$$
 (11)

The cross section $\sigma_{\gamma(Q_1^2)\gamma(Q_2^2)\to J/\psi\rho}$ for the reaction $\gamma(Q_1^2)$ $\gamma(Q_2^2) \to J/\psi\rho^0$ (with J/ψ moving in the direction of $\gamma(Q_1^2)$) depends only weakly on Q_1^2 (Q_2^2) when $Q_1^2 < M_{J/\psi}^2$ ($Q_2^2 < M_{\rho}^2$). On the other hand, the cross section falls rapidly for large Q_i^2 .

This follows from the fact that in processes of this type the cross section is rather a function of $M_V^2 + Q^2$ than of Q^2 alone.

So, for untagged measurements we may account for this property of $\sigma_{\gamma(Q_1^2)\gamma(Q_2^2) \to J/\psi\rho}$ by the standard procedure of taking $\sigma_{\gamma(Q_1^2)\gamma(Q_2^2) \to J/\psi\rho}$ for $Q_1^2 = Q_2^2 = 0$ and imposing the limits $Q_1^2 < M_{J/\psi}^2$ and $Q_2^2 < M_{\rho}^2$. The dependence of the e^+e^- cross section integrated over Q_i^2 on the choice of the upper limits $Q_{i,\max}^2$ is only logarithmic, so there is no need for a more detailed treatment. With this approximation, we may integrate over Q_i^2 to obtain

$$d\sigma_{e^+e^-} = f^{(1)}_{\gamma/e}(z_1) f^{(2)}_{\gamma/e}(z_2) \times \sigma_{\gamma\gamma \to J/\psi\rho}(W^2)|_{Q_1^2 = Q_2^2 = 0} dz_1 dz_2, \quad (12)$$

with

$$f_{\gamma/e}^{(i)}(z) = \frac{\alpha}{2\pi} \left[\frac{1 + (1 - z)^2}{z} \log\left(\frac{Q_{i,\max}^2}{Q_{\min}^2}\right) - 2zm_e^2 \left(\frac{1}{Q_{\min}^2} - \frac{1}{Q_{i,\max}^2}\right) \right], \quad (13)$$

where $Q_{1,\max}^2 = M_{J/\psi}^2$ and $Q_{2,\max}^2 = M_{\rho}^2$. These values of $Q_{i,\max}^2$ are essentially independent of the experimental



Fig. 1. Energy dependence of the cross section $\sigma_{\gamma\gamma\to J/\psi\rho}(W^2)$ for various parameterizations of the vector meson gluonic distribution. The solid line corresponds to the calculations with the GRS-NLO parameterization of the gluon distribution, the dashed one represents the results obtained for the SaS1D parameterization of $xG^V(x,Q^2)$, and the dotted one shows the cross section for the Regge motivated (see (16)) gluon distribution

conditions for the untagged events (if the beam energy is much larger than $Q_{i,\max}$), contrary to the case of the standard measurement of the quasi-real photon structure function in the single anti-tagged events [1].

Let us also introduce a convenient variable to define the corresponding differential cross section:

$$Y = \log \frac{W^2}{M_{J/\psi}^2},\tag{14}$$

i.e., Y corresponds to the $\log(1/x)$. Then it is useful to define

$$\frac{\mathrm{d}\sigma_{e^+e^-}}{\mathrm{d}Y}(Y) = \int \frac{\mathrm{d}\sigma_{e^+e^-}}{\mathrm{d}z_1\mathrm{d}z_2} \delta\left(Y - \log(z_1 z_2 s_{ee}/M_{J/\psi}^2)\right) \times \mathrm{d}z_1\mathrm{d}z_2.$$
(15)

In fact, in the preceding analysis we assumed that J/ψ follows the direction of a particular photon, and if we intend to describe the process in the inclusive way, i.e., irrespectively of the direction of the momentum of the produced mesons, our results should be multiplied by a factor 2. Whenever we use this "direction inclusive" cross section, we shall state it explicitly.

In Fig.1 we display the set of the predicted photon– photon cross sections, $\sigma_{\gamma\gamma\to J/\psi\rho}(W^2)$, obtained with different assumptions concerning the gluon distribution of ρ^0 . Thus, we plot the results for the GRS-NLO parameterization [8] and the SaS1D parameterization [7]. We also include a phenomenological Regge motivated ansatz for $xG^V(x, M_{J/\psi}^2/4)$ by setting for $x < x_0$

$$xG^{V}(x, M_{J/\psi}^{2}/4) = x_{0}G^{V}(x_{0}, M_{J/\psi}^{2}/4) \left(\frac{x_{0}}{x}\right)^{\lambda_{P}}, \quad (16)$$

with $x_0 = 0.1$ and $\lambda_P = 0.25$ in accordance with the HERA data (see also [6]). To obtain the value of $x_0 G^V(x_0,$

¹ It might be also possible to test directly such photon– photon processes at future photon colliders.

Table 1. Total cross section for the process $e^+e^- \rightarrow e^+e^- J/\psi\rho^0$ for the $\gamma\gamma$ energy $W > 10 \,\text{GeV}$. The values are displayed for four e^+e^- collision energies typical for the existing and future accelerators and for three different parameterizations of the gluon distribution

$\sqrt{s_{ee}} \; [\text{GeV}]$	$\sigma(e^+e^- \to e^+e^- J/\psi\rho^0) \text{ [pb]}$		
	GRS	SaS1D	Regge
90	0.9	1.7	0.6
200	3.1	7.0	1.9
500	9.8	27.2	6.0
1000	21	68	13

 $M_{J/\psi}^2/4$) the GRS-NLO parameterization is used. Note that because of the different x-dependencies, i.e. the different values of λ , the correction factors $C_{\rm rp}$ accounting for the real part of the amplitude are different for the three curves. For the rather small $\gamma\gamma$ energy W = 20 GeV, our results for the cross section are larger by a factor between 1.5 (Regge motivated) and 4 (SaS1D) than those obtained in [6]. The steepness of the increase with increasing energy W is strongly dependent upon the choice of the gluon parameterization. The cross sections presented in Fig. 1 might be directly probed at future photon colliders providing constraints on the vector meson structure.

In Fig. 2 we show $d\sigma_{e^+e^-}/dY$ as defined by (15) for two e^+e^- cms energies: $s_{ee}^{1/2} = 200$ (a) and 500 GeV (b) respectively, and for the GRS-NLO, Regge motivated and SaS1D choices of the gluon distribution of the vector meson. The energies are chosen to match LEP2 and future linear collider conditions. The differences in the shape and normalization of the curves in Fig. 2 are large. Even though the normalization factors (e.g. the uncertainty of $B_{J/\psi\rho}$) introduce some uncertainty into formula (8), it still remains possible to distinguish between the models.

In Table 1 the total ("direction inclusive") e^+e^- cross sections for $J/\psi\rho^0$ production, with a cut $W > W_0 =$ 10 GeV (introduced in order to stay in the diffractive regime), are listed for all the three considered models of the gluon distributions and for the energies $s_{ee}^{1/2} = 90$, 200, 500 and 1000 GeV. In particular, we may read from the table that at LEP2 with an integrated luminosity of about $700\,{\rm pb^{-1}}$ per experiment we may expect to have a sizable amount: 1300 to 4900 events. In future colliders, like TESLA [19], the integrated luminosity per year may reach $50 \,\mathrm{fb}^{-1}$ which would correspond to 0.3 to 1.4 million of events. Certainly, the number of interesting events which can be uniquely identified will be severly cut down when the acceptance is taken into account, mainly because J/ψ may be reliably measured only through its leptonic decay products. Another experimental difficulty may arise because, as one may see in the HERA data [10], the total cross section for the proton dissociative photoproduction of J/ψ is larger than for the elastic process. This should also hold for the production of ρ^0 . Besides, there may also occur a hard diffraction of one photon accompanied by a $\gamma \to J/\psi$ transition on the other side. Such processes



Fig. 2a,b. Cross section $d\sigma_{e^+e^-}/dY(Y)$ (see (15)) plotted for three different choices of the gluon distribution and for two e^+e^- collision energies $s_{ee}^{1/2} = 200$ a and 500 GeV b. The solid lines correspond to the calculations with the GRS-NLO parameterization of the gluon distribution, the dashed ones represent the results obtained for the SaS1D parameterization of $xG^V(x, Q^2)$, and the dotted ones show the cross section for the Regge motivated (see (16)) gluon distribution

produce a background for the exclusive processes that we want to focus on; however, their impact on the feasibility of the measurement depends strongly on particular experimental conditions. For instance, at HERA, different classes of events (elastic and proton dissociative) are identified with high efficiency. Of course, the given results may easily be generalized to the other neutral vector meson species, both within the heavy (e.g. ψ', Υ) and light (ω, ϕ) sectors, respectively.

It can also be seen that the measurement proposed by us may be useful to distinguish between the models of the gluon distribution of light vector mesons, i.e., the non-perturbative component of the photonic gluon distribution. In principle, the region of low x would be probed at LEP2 ($x > 10^{-3}$) and at the future linear colliders ($x > 10^{-4}$). Both the shape of the differential cross section $d\sigma_{e^+e^-}/dY$ (see Fig. 2) and the value of the total cross section depend significantly on the model. On top of that, there is this interesting model independent question of how the J/ψ photoproduction off ρ^0 differs from the photoproduction off proton; in particular, if values of the pomeron intercepts $1 + \lambda_P$ characterizing these processes are the same.

To conclude, we studied in detail the exclusive process $\gamma \gamma \rightarrow J/\psi \rho^0$, using as crucial ingredients the vector meson dominance model and the Ryskin picture of elastic vector meson production off a hadron. Measurement of this process could provide for the first time direct information of the gluon distribution in the light vector mesons. It would also constrain the parameterization of the photonic gluon distribution in the low x domain. It may be possible to perform a successful experimental analysis already using the LEP2 data, and at the future linear colliders the number of events would be large enough to perform a thorough study.

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References

- 1. R. Nisius, Phys. Rep. 332, 165 (2000)
- M. Krawczyk, A. Zembrzuski, M. Staszel, hep-ph/0011083 (2000)
- 3. M.G. Ryskin, Z. Phys. C 57, 89 (1993)
- M.G. Ryskin, R.G. Roberts, A.D. Martin, E.M. Levin, Z. Phys. C 76, 231 (1997)

- I.F. Ginzburg, S.L. Panfil, V.G. Serbo, Nucl. Phys. B 296, 569 (1988)
- A. Donnachie, H.G. Dosch, M. Rueter, Phys. Rev. D 59, 074011 (1999)
- G.A. Schuler, T. Sjöstrand, Z. Phys. C 68, 607 (1995); Phys. Lett. B 376, 193 (1996)
- M. Glück, E. Reya, I. Schienbein, Eur. Phys. J. C 10, 313 (1999); Phys. Rev. D 60, 054019 (1999); Erratum ibid. D 62, 019902 (2000)
- B. Badełek, M. Krawczyk, J. Kwieciński, A.M. Staśto, Phys. Rev. D 62, 074021 (2000)
- H1 Collab., T. Ahmed et al., Phys. Lett. B 338, 507 (1994);
 ZEUS Collab., M. Derrick et al., Phys. Lett. B 350, 120 (1995); H1 Collab., S. Aid et al., Nucl. Phys. B 472, 3 (1996); H1 Collab., S. Aid et al., Nucl. Phys. B 468, 3 (1996); ZEUS Collab., J. Breitweg et al., Z. Phys. C 75, 215 (1997); H1 Collab., C. Adloff et al., Phys. Lett. B 483, 23 (2000)
- ZEUS Collab., M. Derrick et al., Phys. Lett. B **356**, 601 (1995); H1 Collab., S. Aid et al., Nucl. Phys. B **463**, 3 (1996); ZEUS Collab., J. Breitweg et al., Eur. Phys. J. C **2**, 247 (1998); H1 Collab., C. Adloff et al., Eur. Phys. J. C **13**, 371 (2000)
- J. Nemchik, N.N. Nikolaev, B.G. Zakharov, Phys. Lett. B 341, 228 (1994); S.J. Brodsky, L. Frankfurt, J.F. Gunion, A.H. Mueller, M. Strikman, Phys. Rev. D 50, 3134 (1994);
 J. Nemchik, N.N. Nikolaev, E. Predazzi, B.G. Zakharov, Phys. Lett. B 374, 199 (1996); L. Frankfurt, W. Koepf, M. Strikman, Phys. Rev. D 57, 512 (1998)
- A.D. Martin, M.G. Ryskin, T. Teubner, Phys. Rev. D 62, 014022 (2000)
- A.D. Martin, M.G. Ryskin, T. Teubner, Phys. Lett. B 454, 339 (1999)
- 15. P. Hoodbhoy, Phys. Rev. D 56, 388 (1997)
- 16. A.V. Radyushkin, Phys. Rev. D 56, 5524 (1997)
- 17. A.D. Martin, M.G. Ryskin, Phys. Rev. D 57, 6692 (1998)
- J. Nemchik, N.N. Nikolaev, E. Predazzi, B.G. Zakharov, V.R. Zoller, J. Exp. Theor. Phys. 86, 1054 (1998)
- 19. R. Brinkmann, DESY-TESLA-99-15 (1999)